Final Exam
Chapters 12 through 16

Name:________________________

Do not write your name on any other page. Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications. Do not approximate answers. Give exact answers. **No calculators are allowed on this exam.**

Show your work!

1. Determine whether the vectors $\mathbf{u} = \mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k}$ and $\mathbf{v} = 2\,\mathbf{i} - \mathbf{j} + \mathbf{k}$ are parallel, orthogonal or neither. (5 points)

2. Determine if the vectors

$$\mathbf{a} = < 6, 3, -1 > \quad \mathbf{b} = < 0, 1, 2 > \quad \mathbf{c} = < 4, -2, 5 >$$

are coplanar. If not, find the volume of the parallelepiped determined by the vectors. (5 points)

3. Find the normal vector to the plane containing the points $P = (2, -1, 2), Q = (-1, 2, -1)$ and $R = (2, 0, 1)$. (8 points)

4. Find the unit vector in the same direction as $\mathbf{v} = 12\ \mathbf{i} - 3\ \mathbf{j} + 4\ \mathbf{k}$. (4 points)

5. Find the arc length of the curve $\mathbf{r}(t) = 12t\ \mathbf{i} + 8t^{3/2}\ \mathbf{j} + 3t^2\ \mathbf{k}$ $(0 \leq t \leq 1)$. (8 points)

6. The position function of a particle is given by $\mathbf{r}(t) =< t^2, 5t, t^2 - 16t >$. When is the speed a minimum?(8 points)

7. Let $z = f(x, y)$ be differentiable and let

$$x = g(t) \quad y = h(t) \quad g(5) = 8 \quad h(5) = 2$$

$$g'(5) = 4 \quad h'(5) = -3 \quad f_x(8, 2) = 6 \quad f_y(8, 2) = -8.$$

Find dz/dt when $t = 5$. (8 points)

8. Find the directional derivative of the function $f(x, y, z) = \sqrt{xyz}$ at the point $(3, 2, 6)$ in the direction of the vector $\mathbf{v} =< -1, -2, 2 >$. (8 points)

9. Determine if $\mathbf{F}(x, y) = (\ln y + 2xy^3)\ \mathbf{i} + (3x^2y^2 + x/y)\ \mathbf{j}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$. (8 points)

10. Use Green's Theorem to evaluate $\int_C xe^{-2x}dx + (x^4 + 2x^2y^2)dy$ where C is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. (8 points)

Pick your poison.

Pick three of the following six problems. Indicate clearly which problems you would like graded. Put your work on the 2 blank sheets following this paper. (10 points each)

1. A particle starts at the point $(-2, 0)$ and moves along the x-axis to the point $(2, 0)$, and then along the semicircle $y = \sqrt{2 - x^2}$ to the starting point. Find the work done on this particle by the force field $\mathbf{F}(x, y) = < x, x^3 + 3xy^2 >$.

2. Let $\mathbf{F}(x, y, z) = (2xz + y^2)\ \mathbf{i} + 2xy\ \mathbf{j} + (x^2 + 3z^2)\ \mathbf{k}$ and C the curve given by $x = t^2$, $y = t + 1, z = 2t - 1$ $(0 \le t \le 1)$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. A lamina occupies the region inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. Find the mass of the lamina and set up the equations to find the center of mass if the density at any point is inversely proportional to its distance from the origin. (i.e. $\rho(x, y) = \frac{K}{\sqrt{x^2+y^2}}$)

4. Let X and Y be variables with joint density function given by

$$f(x, y) = \begin{cases} .1e^{-(.2x+.5y)} & x, y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

   Find $P(X \ge 2, Y \ge 4)$.

5. Evaluate $\iiint_E (x^3 + xy^2) dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.

6. Evaluate $\iiint_E xyz dV$, where E lies between $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 16$ and above the cone $\phi = \pi/3$.
   <u>**Hint:**</u> Use Fubini's Theorem.